

Kenmore-Town of Tonawanda UFSD

We educate, prepare, and inspire all students to achieve their highest potential

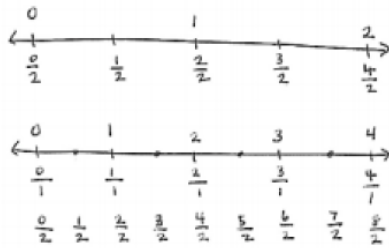


Grade 3 Module 5 Parent Handbook

Fractions as Numbers on the Number Line

In this 35-day module, students extend and deepen 2nd grade practice with “equal shares” to understanding fractions as equal partitions of a whole. They formalize their knowledge as they work with area models and the number line.

Students will learn to partition number lines into fractional parts, renaming whole numbers as fractions.



In this activity, students specify and partition a whole into equal parts, identifying and counting unit fractions by folding fraction strips.



What Came Before this Module:

Students explored area as an attribute of two-dimensional figures and related it to their prior work with multiplication.

What Comes After this Module:

In Module 6, students will begin work on data collection and representation. Specifically, students will generate and analyze categorical and measurement data.

Key Terms and Ideas

New Terms:

Unit fraction- fractions with numerator of 1

Non-unit fraction- fractions with numerators other than 1
Fractional unit- half, third, fourth, etc.

Equal parts- parts with equal measurements

Unit interval- the interval from 0 to 1, measured by length

Equivalent fraction- fractions that are the same size, or the same point on a number line

Copies- refers to the number of unit fractions in one whole

Terms and Symbols to Review:

Number Line

Arrays

Equal Shares

Whole

Fraction

Partition

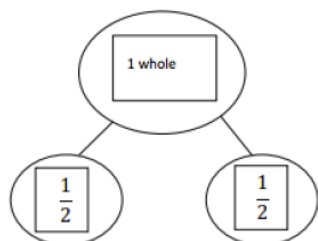
=, <, >

+ How you can help at home:

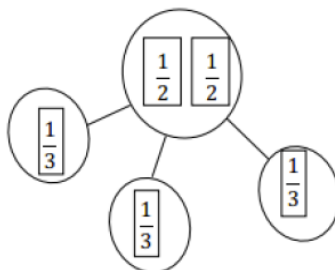
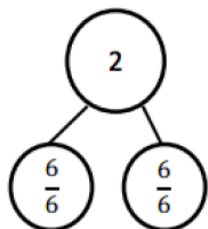
- ⇒ Continue to review multiplication and division math facts with your student
- ⇒ Help students practice partitioning household items (pieces of paper, portions of food, a pack of crayons, etc.) into equal parts

Key Common Core Standards:

- **Develop understanding of fractions as numbers**
 - Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$
 - Understand a fraction as a number on the number line; represent fractions on a number line diagram.
 - Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
- **Reason with shapes and their attributes**
 - Partition shapes into parts with equal areas



Various number bonds students will encounter in this module



Spotlight on Math Models:

Number Bonds

You will often see this mathematical representation in *A Story of Units*.

A Story of Units has several key mathematical “models” that will be used throughout a student’s elementary years.

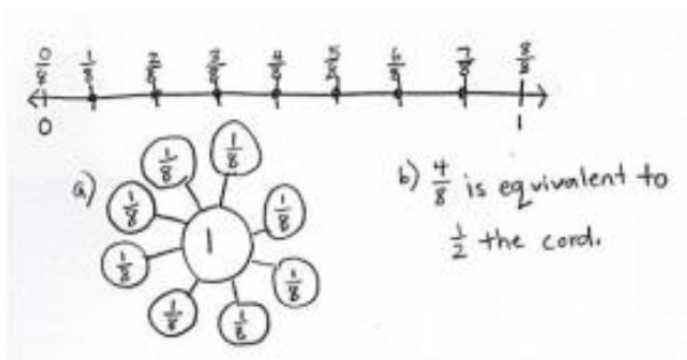
The number bond is a pictorial representation of part/part/whole relationships showing that smaller numbers (the parts) make up larger numbers (the whole). The number bond is a key model for showing students how to both take apart (decompose) and put together (compose) numbers.

Students become familiar with number bonds in Kindergarten, and they are used repeatedly throughout the grades in various situations. In Grade 3, students compose fractional numbers using number bonds as a powerful tool to see the unit fractions that make up a whole number. They will also use number bonds to decompose whole numbers greater than 1 into fractional parts.

Module 5 Sample Problem
(Example taken from Lesson 22)

Mr. Ramos wants to nail the TV cord against the wall so no one trips. He puts 7 nails equally spaced along the cord. Draw a number line representing the cord. Label it from 0 at the start of the cord to 1 at the end. Put a mark where Mr. Ramos puts each nail with a fraction.

- Build a number bond with unit fractions to 1 whole.
- Write the fraction of the nail that is equivalent to $\frac{1}{2}$ the cord.



Fractions as Numbers on the Number Line

OVERVIEW

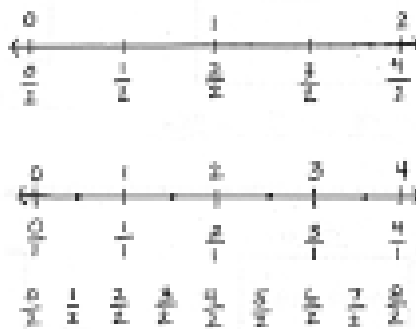
In this 35-day module, students extend and deepen Grade 2 practice with equal shares to understanding fractions as equal partitions of a whole (**2.G.3**). Their knowledge becomes more formal as they work with area models and the number line. Throughout the module, students have multiple experiences working with the Grade 3 specified fractional units of halves, thirds, fourths, sixths, and eighths. To build flexible thinking about fractions, students are exposed to additional fractional units such as fifths, ninths, and tenths.

Topic A opens Module 5 with students actively partitioning different models of wholes into equal parts (e.g., concrete models and drawn pictorial area models on paper). They identify and count equal parts as *1 half*, *1 fourth*, *1 third*, *1 sixth*, and *1 eighth* in unit form before introduction to the unit fraction $\frac{1}{b}$ (**3.NF.1**). In Topic B, students compare and make copies of unit fractions to build non-unit fractions. They understand unit fractions as the basic building blocks that compose other fractions (**3.NF.3d**), which parallels the understanding that the number 1 is the basic building block of whole numbers. In Topic C, students practice comparing unit fractions to fraction strips. They specify the whole and label fractions in relation to the number of equal parts in that whole (**3.NF.3d**).



Compare unit fractions using fraction strips.

Students transfer their work to the number line in Topic D. They begin by using the interval from 0 to 1 as the whole. Continuing beyond the first interval, they partition, place, count, and compare fractions on the number line (**3.NF.2a**, **3.NF.2b**, **3.NF.3d**). In Topic E, they notice that some fractions with different units are placed at the exact same point on the number line, and therefore, are equal (**3.NF.3a**). For example, $\frac{12}{24}$, $\frac{24}{36}$, and $\frac{36}{48}$ are equivalent fractions (**3.NF.3b**). Students recognize that whole numbers can be written as fractions, as exemplified on the number lines to the left (**3.NF.3c**).



Topic F concludes the module with comparing fractions that have the same numerator. As students compare fractions by reasoning about their size, they understand that fractions with the same numerator and a larger denominator are actually smaller pieces of the whole (**3.NF.3d**). Topic F leaves students with a new method for precisely partitioning a number line into unit fractions of any size without using a ruler.

Terminology

New or Recently Introduced Terms

- Copies (refers to the number of unit fractions in 1 whole)
- Equivalent fractions (fractions that name the same size or the same point on the number line)
- Fractional unit (half, third, fourth, etc.)
- Non-unit fraction (fraction with numerator other than 1)
- Unit fraction (fraction with numerator 1)
- Unit interval (the interval from 0 to 1, measured by length)

Familiar Terms and Symbols²

- =, <, > (equal, less than, greater than)
- Array (arrangement of objects in rows and columns)
- Equal parts (parts with equal measurements)
- Equal shares (pieces of a whole that are the same size)
- Fraction (e.g., 13, 23, 33, 43)
- Half of, one third of, one fourth of, etc. (12, 13, 14, 16, 18)
- Halves, thirds, fourths, sixths, eighths (12, 13, 14, 16, 18)
- Number line
- Partition (divide a whole into equal parts)
- Whole (e.g., 2 halves, 3 thirds, etc.)

Suggested Tools and Representations

- 1 m length of yarn
- 12" × 1" strips of yellow construction paper
- 1-liter beaker (optional)
- 2" × 6" strips of brown construction paper
- 200 g ball of clay or play dough
- 4 14" × 1" paper strips
- 4" × 4" orange squares
- Arrays
- Clear plastic cups
- Concrete fraction models (e.g., water, string, clay)
- Food coloring (to color water)
- Fraction strips (made from paper, used to fold and model parts of a whole. See example to the right.)
- Number line
- Pictorial fraction model (e.g., drawing of a circle or square)
- Rectangular- and circular-shaped paper
- Rulers
- Sets of <, >, = cards
- Shapes partitioned into fractional parts
- Tape diagram



Fraction strips

Grade 3 Module 5 Topic A

Partitioning a Whole into Equal Parts

Focus Standard:

3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.

Instructional Days Recommended: 4

In Topic A, students partition a whole using a ruler to precisely measure equal parts. They then see how cups can be used to measure equal parts of water. From there, students are invited to fold fraction strips and then estimate to draw pictorial models. The topic culminates in an exploration, wherein they model a designated fraction with a meter string, 12 ounces of water, 200 grams of clay, a 4" × 4" square, a 12" × 1" strip, and a 6" × 2" strip. Students then tour the fraction displays created by their peers and analyze their observations. They specify that the whole contains a certain number of equal parts.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 1

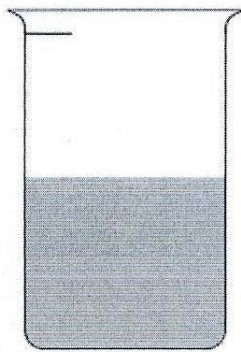
Objective: Specify and partition a whole into equal parts, identifying and counting unit fractions using concrete models.

Homework Key

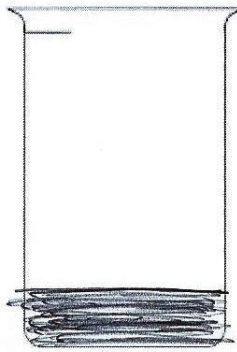
1. Answer provided; 1 fifth shaded; 1 sixth shaded
2. 1 third; 1 fourth; 1 seventh
3. Halves are shown; thirds are shown; sixths are shown
4. Lines drawn to show halves; lines drawn to show fourths; lines drawn to show eighths
5. Lines drawn to show sixths; lines drawn to show thirds
6. 1 sixth; picture drawn to show 6 equal parts
7. 6 grams

Homework Samples

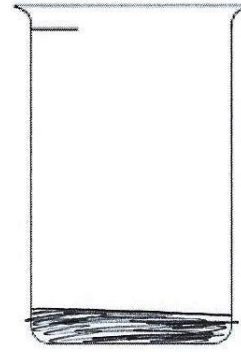
1. A beaker is considered full when the liquid reaches the fill line shown near the top. Estimate the amount of water in the beaker by shading the drawing as indicated. The first one is done for you.



1 half

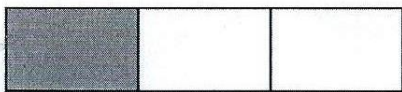


1 fifth



1 sixth

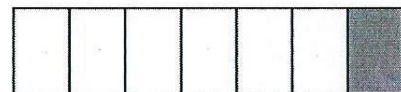
2. Danielle cut her candy bar into equal pieces as shown in the rectangles below. In the blanks below, name the fraction of candy bar represented by the shaded part.



1 third



1 fourth



1 seventh

Lesson 2

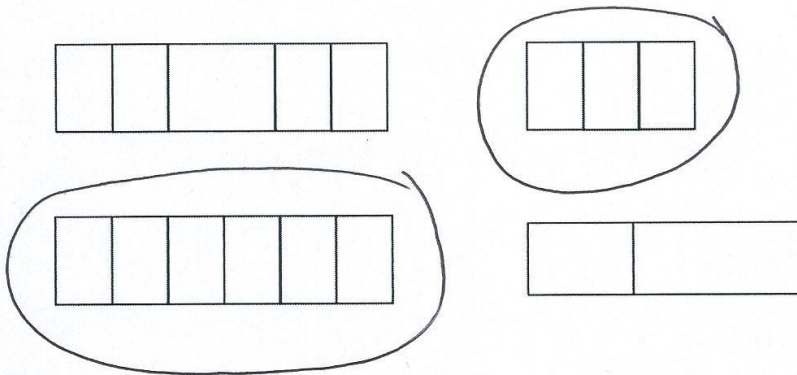
Objective: Specify and partition a whole into equal parts identifying and counting unit fractions by folding fraction strips.

Homework Key

1. Second and third strips circled
2. a. 2, 1
b. 3, 1
c. 5, 1
d. 14, 7
3. Answers will vary.
4. a. 1 eighth
b. 5 eighths

Homework Samples

1. Circle the strips that are cut into equal parts.

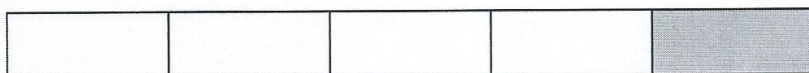


- 2.

a. There are 2 equal parts in all. 1 is shaded.



b. There are 3 equal parts in all. 1 is shaded.



c. There are 5 equal parts in all. 1 is shaded.



d. There are 14 equal parts in all. 7 are shaded.

Lesson 3

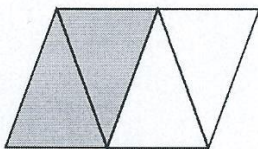
Objective: Specify and partition a whole into equal parts, identifying and counting unit fractions by drawing pictorial area models.

Homework Key

1. Fifths; 4 fifths
Sixths; 3 sixths
Halves; 0 halves
2. Answers will vary.
3. Calendar drawn and divided into 12 equal parts; 1 twelfth

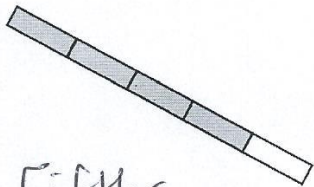
Homework Sample

1. Each shape is a whole divided into equal parts. Name the fractional unit, and then count and tell how many of those units are shaded. The first one is done for you.



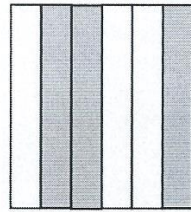
Fourths

2 fourths are shaded.



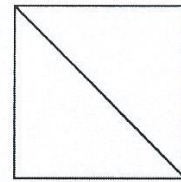
Fifths

4 fifths are shaded.



Sixths

3 sixths are shaded.



Halves

0 halves are shaded

Lesson 4

Objective: Represent and identify fractional parts of different wholes.

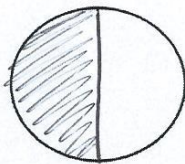
Homework Key

1. Lines drawn to show halves for each figure; each figure shaded to show 1 half
2. Lines drawn to show fourths for each figure; each figure shaded to show 1 fourth
3. Lines drawn to show thirds for each figure; each figure shaded to show 1 third
4. Fractions matched to equivalent shape

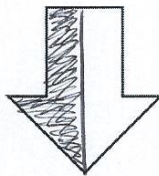
Homework Sample

Each shape is 1 whole. Estimate to equally partition the shape and shade to show the given fraction.

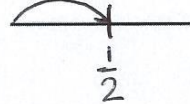
1. 1 half



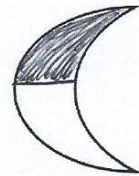
A



B



C



D

Grade 3 Module 5 Topic B

Unit Fractions and Their Relation to the Whole

Focus Standard:

3.NF.1 Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

Instructional Days Recommended: 5

In Topic A, students divided a given whole into equal parts to create fractional units (halves, thirds, fourths, etc.). Now, they associate one of the fractional units with a number called the unit fraction ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc.). This sets the foundation for students eventually understanding that a fraction is a number. Like any number, it corresponds to a point on the real number line and can be written in unit form or fraction form (e.g., 1 half or $\frac{1}{2}$).

An advantage of the term *fractional unit* is that it distinguishes the nature of the equal parts generated by partitioning a whole from the whole number division students studied in Modules 1 and 3. In Topic B, to avoid confusion, the term *fractional unit* is mostly replaced by the term *equal part*. The equal part is represented by the unit fraction. Students recognize that any non-unit fraction is composed of multiple copies of a unit fraction. They use number bonds to represent this. In particular, students construct fractions greater than 1 using multiple copies of a given unit fraction.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 5

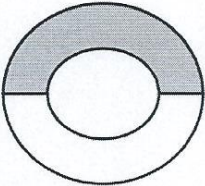
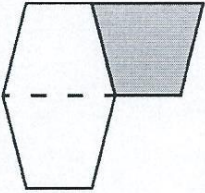
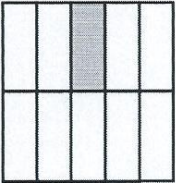
Objective: Partition a whole into equal parts and define the equal parts to identify the unit fraction numerically.

Homework Key

1.
 - a. 2, 1, 1 half, $\frac{1}{2}$
 - b. 3, 1, 1 third, $\frac{1}{3}$
 - c. 10, 1, 1 tenth, $\frac{1}{10}$
 - d. 5, 1, 1 fifth, $\frac{1}{5}$
 - e. 4, 1, 1 fourth, $\frac{1}{4}$
2. No; explanations will vary.
3. Lines drawn to show fourths
4. Rectangles drawn and labeled to show $\frac{1}{7}$ and $\frac{1}{10}$ (4. Continued) explanations will vary

Homework Sample

1. Fill in the chart. Each image is one whole.

	Total Number of Equal Parts	Total Number of Equal Parts Shaded	Unit Form	Fraction Form
a. 	2	1	1 half	$\frac{1}{2}$
b. 	3	1	1 third	$\frac{1}{3}$
c. 	10	1	1 tenth	$\frac{1}{10}$

Lesson 6

Objective: Build non-unit fractions less than one whole from unit fractions.

Homework Key

1. a. $\frac{2}{3}$

b. $\frac{5}{7}$

c. $\frac{3}{5}$

d. $\frac{2}{8}$

2. a. $\frac{1}{6}$

b. $\frac{5}{6}$

3. a. $4, 3, \frac{1}{4}, \frac{3}{4}$

b. $9, 6, \frac{1}{9}, \frac{6}{9}$

c. $7, 4, \frac{1}{7}, \frac{4}{7}$

d. $6, 3, \frac{1}{6}, \frac{3}{6}$

Homework Samples

1. Complete the number sentence. Estimate to partition each strip equally, write the unit fraction inside each unit, and shade the answer.

Sample:

3 fourths = $\frac{3}{4}$



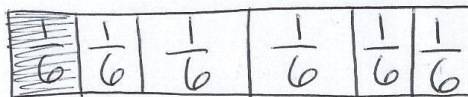
a. 2 thirds = $\frac{2}{3}$



2. Mr. Abney bought 6 kilograms of rice. He cooked 1 kilogram of it for dinner.

- a. What fraction of the rice did he cook for dinner?

$\frac{1}{6}$ kilogram



- b. What fraction of the rice was left?

$\frac{5}{6}$ kilogram

Lesson 7


Objective: Identify and represent shaded and non-shaded parts of one whole as fractions.

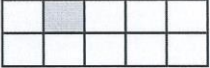
Homework Key

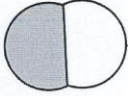
1. 3 fourths
2. 9 tenths
3. 1 half
4. 2 thirds
5. 6 sevenths
6. 4 fifths
7. 10 elevenths
8. 5 sixths
9. $\frac{1}{5}, \frac{4}{5}, \frac{1}{12}, \frac{11}{12}$
10. 3 fourths; picture drawn and labeled to show $\frac{1}{4}$ finished and $\frac{3}{4}$ unfinished
11. 1 eighth; picture drawn and labeled to show $\frac{1}{8}$ uneaten and $\frac{7}{8}$ eaten


Homework Samples

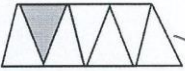
Whisper the fraction of the shape that is shaded. Then, match the shape to the amount that is not shaded.


1.  9 tenths

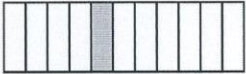
2.  4 fifths


3.  10 elevenths

4.  5 sixths

5.  1 half

6.  2 thirds

7.  3 fourths

8.  6 sevenths

Lesson 8

Objective: Represent parts of one whole as fractions with number bonds.

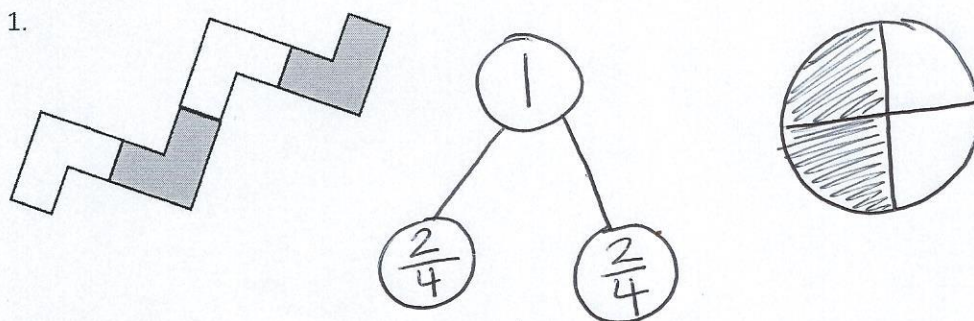
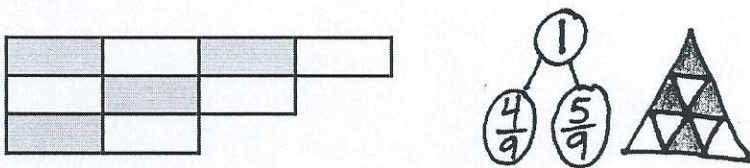
Homework Key

- Number bond showing $\frac{2}{4}$ and $\frac{2}{4}$ equals 1 whole; second visual model drawn
- Number bond showing $\frac{5}{7}$ and $\frac{2}{7}$ equals 1 whole; second visual model drawn
- Number bond showing $\frac{4}{5}$ and $\frac{1}{5}$ equals 1 whole; second visual model drawn
- Number bond showing $\frac{0}{8}$ and $\frac{8}{8}$ equals 1 whole; second visual model drawn
- Number bond showing $\frac{2}{3}$ and $\frac{1}{3}$ equals 1 whole; $\frac{2}{3}$ decomposed showing 2 units of $\frac{1}{3}$
 - Number bond showing $\frac{4}{5}$ and $\frac{1}{5}$ equals 1 whole; $\frac{4}{5}$ decomposed showing 4 units of $\frac{1}{5}$
 - Number bond showing $\frac{3}{5}$ and $\frac{2}{5}$ equals 1 whole; $\frac{3}{5}$ decomposed showing 3 units of $\frac{1}{5}$; $\frac{2}{5}$ decomposed showing 2 units of $\frac{1}{5}$
- Drawing showing 3 equal parts; 2 thirds shaded; number bond showing $\frac{2}{3}$ and $\frac{1}{3}$ equals 1 whole; $\frac{2}{3}$

Homework Samples

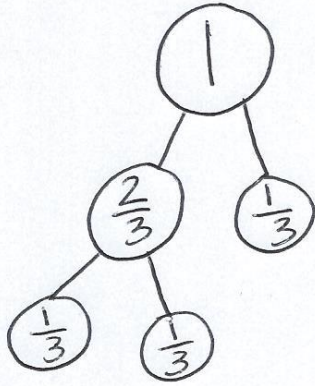
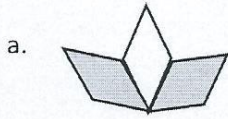
Show a number bond representing what is shaded and unshaded in each of the figures. Draw a different visual model that would be represented by the same number bond.

Sample:



Lesson 8 (continued)

5. Draw a number bond with 2 parts showing the shaded and unshaded fractions of each figure. Decompose both parts of the number bond into unit fractions.



Lesson 9

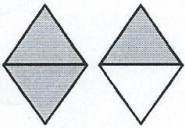
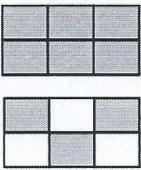
Objective: Build and write fractions greater than one whole using unit fractions.

Homework Key

- Answer provided
 - $\frac{1}{6}, 9, \frac{9}{6}$
 - $\frac{1}{4}, 15, \frac{15}{4}$
 - $\frac{1}{2}, 6, \frac{6}{2}$
 - $\frac{1}{3}, 4, \frac{4}{3}$
 - $\frac{1}{3}, 4, \frac{4}{3}$
- Each whole partitioned into thirds; 5 thirds shaded; $\frac{5}{3}$
 - Each whole partitioned into thirds; 9 thirds shaded; 9 thirds
- 2 equivalent wholes drawn; each whole partitioned into 4 equal pieces; 5 pieces shaded
 - $\frac{5}{4}$

Homework Samples

- Each shape represents 1 whole. Fill in the chart.

	Unit Fraction	Total Number of Units Shaded	Fraction Shaded
a. Sample: 	$\frac{1}{2}$	3	$\frac{3}{2}$
b. 	$\frac{1}{6}$	9	$\frac{9}{6}$

- Estimate to draw and shade units on the fraction strips. Solve.

Sample:

$$7 \text{ fourths} = \frac{7}{4}$$



- 5 thirds = $\frac{5}{3}$



Grade 3 Module 5 Topic C

Comparing Unit Fractions and Specifying the Whole

Focus Standards:

- 3.NF.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
- d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Instructional Days Recommended: 4

Students practiced identifying and labeling unit and non-unit fractions in Topic B. Now, in Topic C, they begin by comparing unit fractions. Using fraction strips, students recognize that, when the same whole is folded into more equal parts, each part is smaller. Next, using real-life examples and area models, students understand that, when comparing fractions, the whole must be the same size. Next, students create corresponding wholes based on a given unit fraction using similar materials to those in Lesson 4's exploration: clay, yarn, two rectangles, and a square. They conduct a *museum walk* to study the wholes, identifying the unit fractions and observing part-whole relationships. Finally, students learn that redefining the whole can change the unit fraction that describes the shaded part.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 10

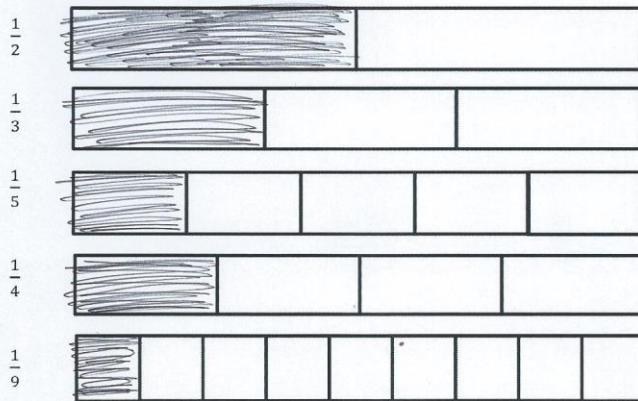
Objective: Compare unit fractions by reasoning about their size using fractions strips.

Homework Key

- | | |
|---|--|
| 1. Specified fractional unit shaded in each strip | 3. More water; explanations will vary. |
| 2. a. Greater than | 4. a. > |
| b. Less than | b. < |
| c. Less than | c. = |
| d. Greater than | d. < |
| e. Less than | e. < |
| f. Less than | f. = |
| g. Greater than | g. > |
| h. Greater than | 5. Answers will vary. |

Homework Samples

1. Each fraction strip is 1 whole. All the fraction strips are equal in length. Color 1 fractional unit in each strip. Then, answer the questions below.



2. Circle *less than* or *greater than*. Whisper the complete sentence.

- | | |
|---|---|
| a. $\frac{1}{2}$ is <u>greater than</u> $\frac{1}{3}$ | b. $\frac{1}{9}$ is <u>less than</u> $\frac{1}{2}$ |
| c. $\frac{1}{4}$ is <u>less than</u> $\frac{1}{2}$ | d. $\frac{1}{4}$ is <u>greater than</u> $\frac{1}{9}$ |
| e. $\frac{1}{5}$ is <u>less than</u> $\frac{1}{3}$ | f. $\frac{1}{5}$ is <u>less than</u> $\frac{1}{4}$ |
| g. $\frac{1}{2}$ is <u>greater than</u> $\frac{1}{5}$ | h. 6 fifths is <u>greater than</u> 3 thirds |

6 fifths is greater than 1 and 3 thirds equals 1

Lesson 11



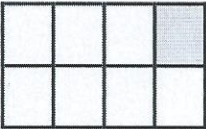
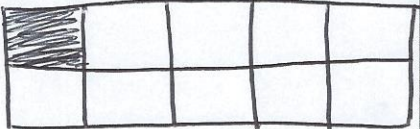
Objective: Compare unit fractions with different-sized models representing the whole.

Homework Key

1. $\frac{1}{8}$; answers will vary.
2. $\frac{1}{4}$; answers will vary.
3. $\frac{1}{10}$; answers will vary.
4. $\frac{1}{9}$; answers will vary.
5. Answers will vary; $\frac{1}{2}$
6. Answers will vary; $\frac{1}{4}$
7. Answers will vary; $\frac{1}{12}$
8. Answers will vary.
9. a. Explanations will vary.
b. Explanations will vary.

Homework Sample

Label the unit fraction. In each blank, draw and label the same whole with a shaded unit fraction that makes the sentence true. There is more than 1 correct way to make the sentence true.

<p>Sample:</p> <p>$\frac{1}{3}$</p> 	<p>is less than</p>	<p>$\frac{1}{2}$</p> 
<p>1.</p> <p>$\frac{1}{8}$</p> 	<p>is greater than</p>	<p>$\frac{1}{10}$</p> 

Lesson 12

Objective: Specify the corresponding whole when presented with one equal part.

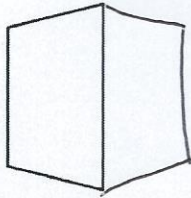
Homework Key

1. Picture representing 2 halves
2. Picture representing 6 sixths
3. Picture representing 3 thirds
4. Picture representing 4 fourths
5. Answer provided
6. Picture representing 2 halves; number bond showing 2 units of $\frac{1}{2}$ equals 1 whole
7. Picture representing 5 fifths; number bond showing 5 units of $\frac{1}{5}$ equals 1 whole
8. Picture representing 7 sevenths; number bond showing 7 units of $\frac{1}{7}$ equals 1 whole
9. No, explanations will vary.

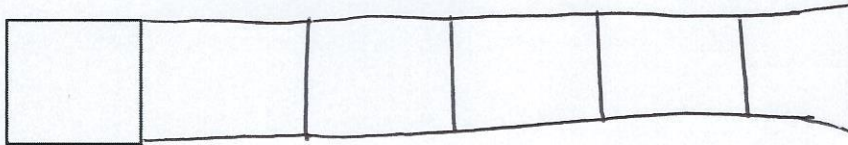
Homework Samples

Each shape represents the given unit fraction. Estimate to draw a possible whole.

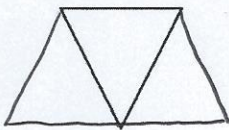
1. $\frac{1}{2}$



2. $\frac{1}{6}$



3. 1 third



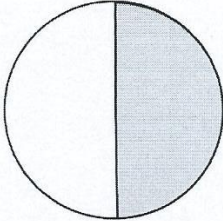
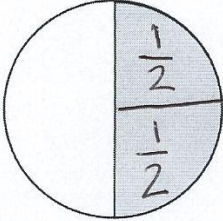
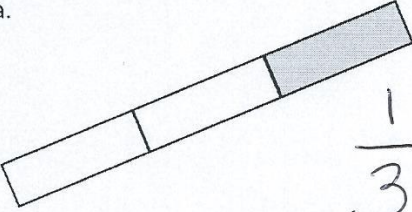
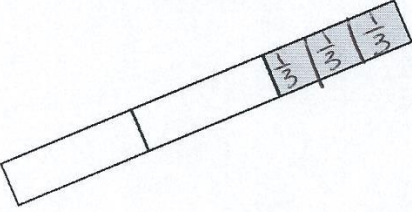
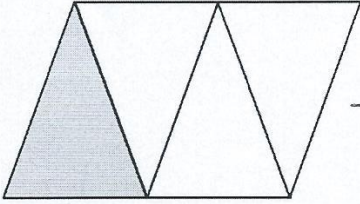
Lesson 13

Objective: Identify a shaded fractional part in different ways depending on the designation of the whole.

Homework Key

1. a. $\frac{1}{2}$
b. Shaded part divided to show $\frac{1}{2}$
2. a. $\frac{1}{3}$
b. Shaded part divided to show $\frac{1}{3}$
3. a. $\frac{1}{4}$
b. Shaded part divided to show $\frac{1}{4}$
4. a. $\frac{1}{5}$
b. Shaded part divided to show $\frac{1}{5}$
5. a. B
b. A
c. 3; 2
d. 3; number bond showing 3 units of $\frac{1}{3}$ equals 1 whole
e. 2; number bond showing 2 units of $\frac{1}{2}$ equals 1 whole
6. Strings drawn correctly

Homework Samples

The shape represents 1 whole. Write a fraction to describe the shaded part.	The shaded part represents 1 whole. Divide 1 whole to show the same unit fraction you wrote in Part (a).
1. a. $\frac{1}{2}$ 	b. 
2. a.  $\frac{1}{3}$	b. 
3. a.  $\frac{1}{4}$	

Grade 3 Module 5 Topic D

Fractions on the Number Line

Focus Standards:

- 3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram.
- Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.
 - Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
- 3.NF.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
- Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.*
 - Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, $<$, and justify the conclusions, e.g., by using a visual fraction model.

Instructional Days Recommended: 6

In Topic C, students compared unit fractions and explored the importance of specifying the whole when doing so. In Topic D, they apply their learning to the number line. Number bonds and fraction strips serve as bridges into this work. Students see intervals on the number line as wholes. They initially measure equal lengths between 0 and 1 with their fraction strips. They then work with number lines that have endpoints other than 0 and 1 or include multiple whole number intervals. This naturally transitions into comparing fractions with the same denominator, as well as fractional numbers and whole numbers on the number line. As students compare, they reason about the size of fractions and contextualize their learning within real-world applications.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 14

Objective: Place fractions on a number line with endpoints 0 and 1.

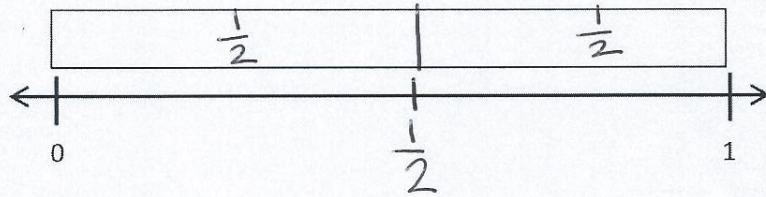
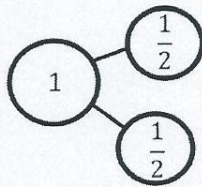
Homework Key

- Already completed number bond; fraction strip is partitioned and labeled correctly to show halves; number line partitioned and labeled correctly from $\frac{0}{2}$ to $\frac{2}{2}$
 - Number bond is drawn correctly to show 8 units of $\frac{1}{8}$; fraction strip is partitioned and labeled correctly to show eighths; number line partitioned and labeled correctly from $\frac{0}{8}$ to $\frac{8}{8}$
 - Number bond is drawn correctly to show 5 units of $\frac{1}{5}$; fraction strip is partitioned and labeled correctly to show fifths; number line partitioned and labeled correctly from $\frac{0}{5}$ to $\frac{5}{5}$
- Yes
- 9 seeds
 - 36 seeds
 - Number line is drawn and partitioned correctly to show ninths

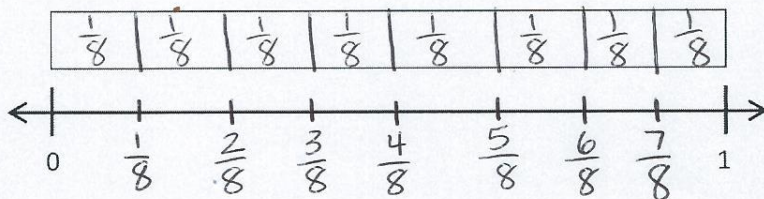
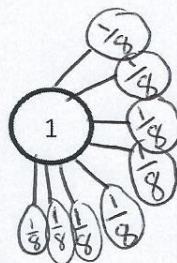
Homework Sample

- Draw a number bond for each fractional unit. Partition the fraction strip to show the unit fractions of the number bond. Use the fraction strip to help you label the fractions on the number line. Be sure to label the fractions at 0 and 1.

a. Halves



b. Eighths



Lesson 15

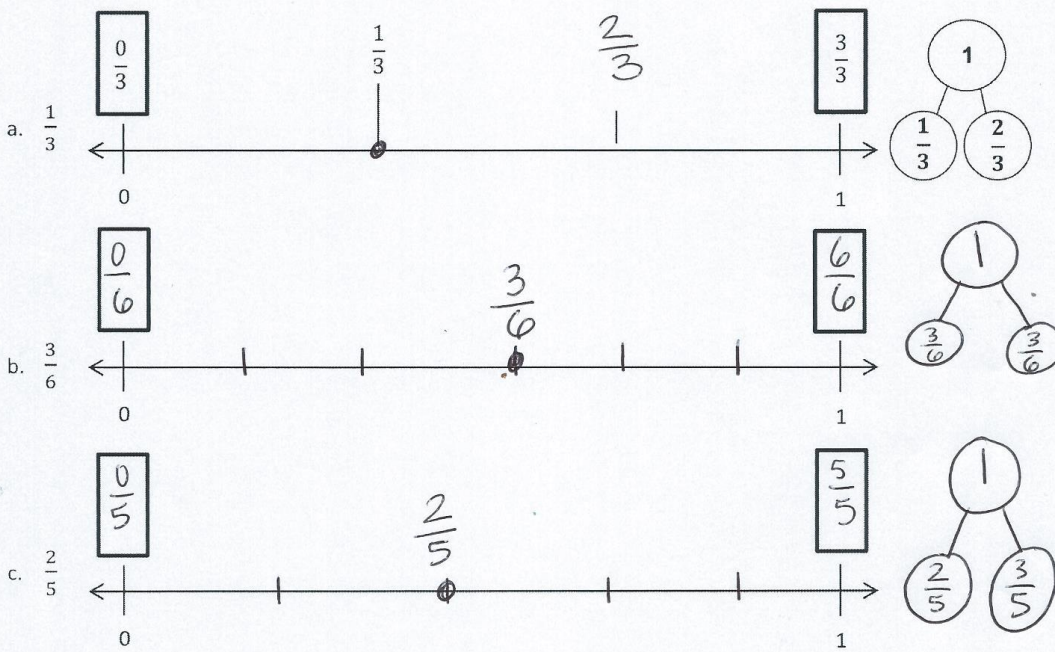
Objective: Place any fraction on a number line with endpoints 0 and 1.

Homework Key

1.
 - a. Answer provided
 - b. Number line partitioned into sixths and labeled correctly with $\frac{0}{6}, \frac{3}{6}, \frac{6}{6}$; number bond showing $\frac{3}{6}$ and $\frac{3}{6}$ equals 1 whole
 - c. Number line partitioned into fifths and labeled correctly with $\frac{0}{5}, \frac{2}{5}, \frac{5}{5}$; number bond showing $\frac{2}{5}$ and $\frac{3}{5}$ equals 1 whole
 - d. Number line partitioned into tenths and labeled correctly with $\frac{0}{10}, \frac{7}{10}, \frac{10}{10}$; number bond showing $\frac{7}{10}$ and $\frac{3}{10}$ equals 1 whole
 - e. Number line partitioned into sevenths and labeled correctly with $\frac{0}{7}, \frac{3}{7}, \frac{7}{7}$; number bond showing $\frac{3}{7}$ and $\frac{4}{7}$ equals 1 whole
2.
 - a. Henry: $\frac{5}{10}$; Ben: $\frac{9}{10}$; Tina: $\frac{2}{10}$
 - b. Number line partitioned into tenths; $\frac{2}{10}, \frac{5}{10}, \frac{9}{10}$ placed correctly on the number line
3.
 - a. Number line drawn with 0 and 1 labeled correctly; fraction strip used appropriately to partition and label number line to show eighths; number line labeled correctly from $\frac{0}{8}$ to $\frac{8}{8}$
 - b. Touched and counted each fraction from 0 eighths to 8 eighths

Homework Sample

1. Estimate to label the given fractions on the number line. Be sure to label the fractions at 0 and 1. Write the fractions above the number line. Draw a number bond to match your number line. The first one is done for you.



Lesson 16

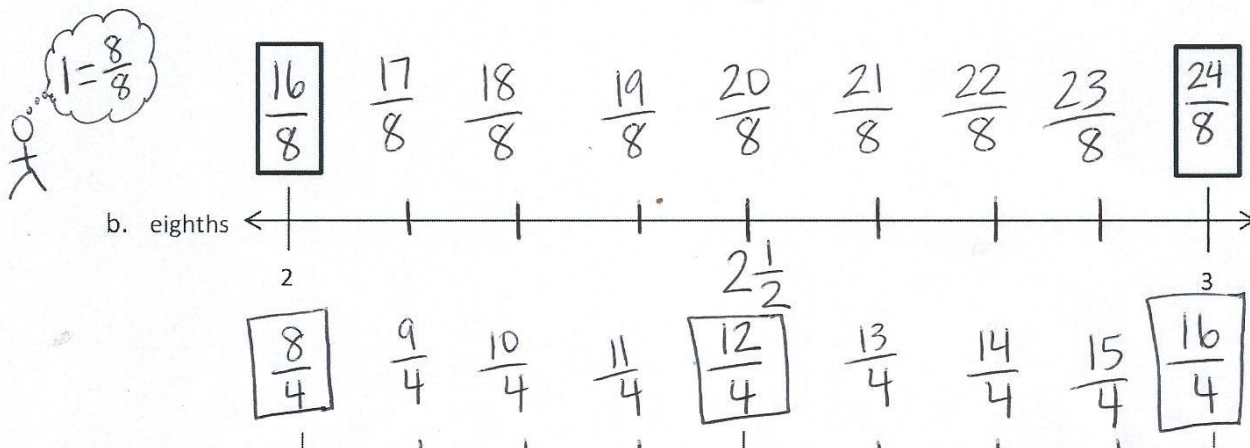
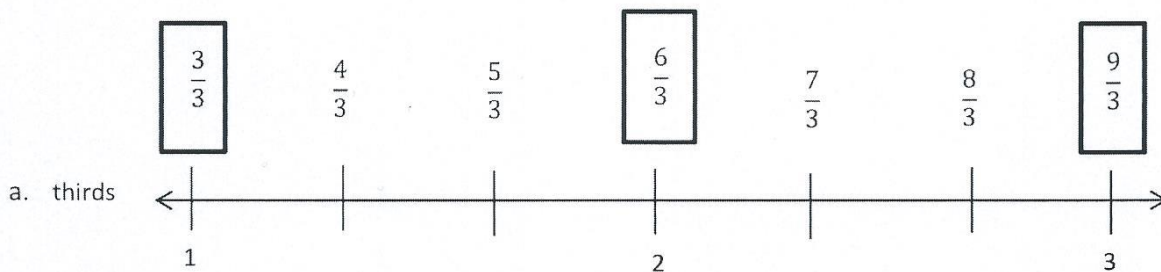
Objective: Place whole number fractions and fractions between whole numbers on the number line.

Homework Key

- Answer provided
 - Number line partitioned into eighths and labeled; $\frac{16}{8}, \frac{24}{8}$ boxed
 - Number line partitioned into fourths and labeled; $\frac{8}{4}, \frac{12}{4}, \frac{16}{4}$ boxed
 - Number line partitioned into halves and labeled; $\frac{6}{2}, \frac{8}{2}, \frac{10}{2}$ boxed; 4 labeled below $\frac{8}{2}$
 - Number line partitioned into fifths and labeled; $\frac{30}{5}, \frac{35}{5}, \frac{40}{5}, \frac{45}{5}$ boxed; 7 labeled below $\frac{35}{5}$, 8 labeled below $\frac{40}{5}$
- Number line partitioned into sixths and labeled; $\frac{18}{6}, \frac{24}{6}, \frac{30}{6}$ boxed
- Number line partitioned into halves and labeled; $\frac{8}{2}, \frac{10}{2}, \frac{12}{2}, \frac{14}{2}$ boxed
- Number line with endpoints 0 and 3; wholes labeled; number line partitioned and labeled

Homework Sample

- Estimate to equally partition and label the fractions on the number line. Label the wholes as fractions, and box them. The first one is done for you.



Lesson 17

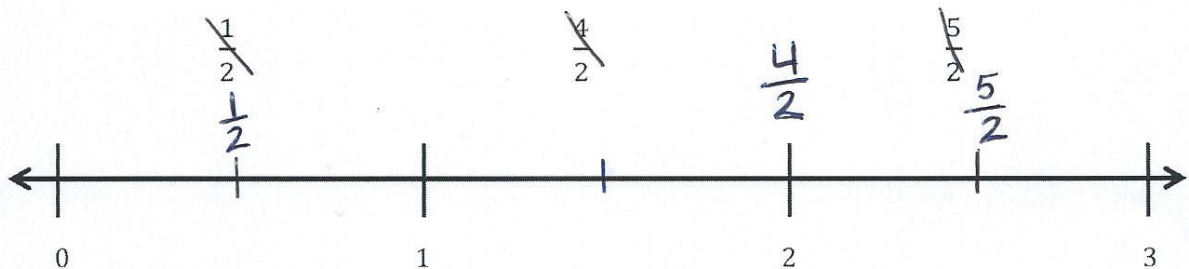
Objective: Practice placing various fractions on the number line.

Homework Key

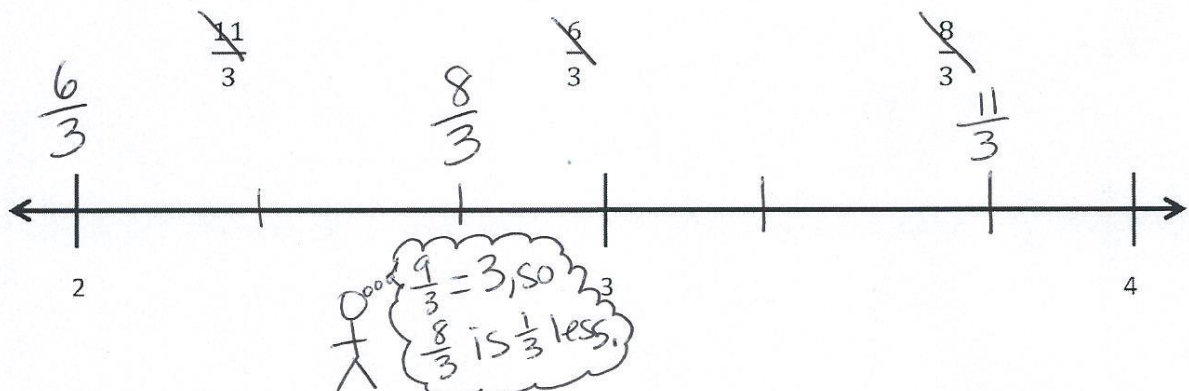
1. Number line partitioned into halves; given fractions located and labeled
2. Number line partitioned into thirds; given fractions located and labeled
3. Number line partitioned into fourths; given fractions located and labeled
4. Number line drawn with endpoints 0 km to 4 km, partitioned correctly into thirds; $\frac{0}{3}$ (0) km, $\frac{12}{3}$ (4) km, $\frac{4}{3}$ km, $\frac{10}{3}$ km located and labeled
5. Yes; number line partitioned into fourths; $\frac{19}{4}$ ft. located and labeled; number line showing $\frac{19}{4}$ ft. is smaller than 5 ft.

Homework Sample

1. Locate and label the following fractions on the number line.



2. Locate and label the following fractions on the number line.



Lesson 18

Objective: Compare fractions and whole numbers on the number line by reasoning about their distance from 0.

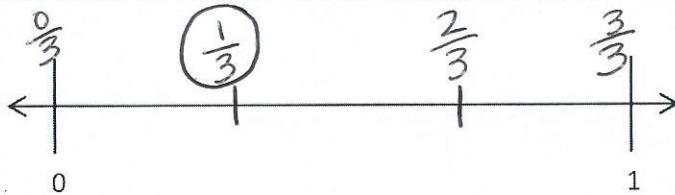
Homework Key

- Number line partitioned into thirds; $\frac{1}{3}$ and $\frac{2}{3}$ placed; $\frac{1}{3}$ circled; $<$
- Number line partitioned into sixths; $\frac{4}{6}$ and $\frac{1}{6}$ placed; $\frac{1}{6}$ circled; $>$
- Number line partitioned into fourths and eighths; $\frac{1}{4}$ and $\frac{1}{8}$ placed; $\frac{1}{8}$ circled; $>$
- Number line partitioned into fifths and tenths; $\frac{4}{5}$ and $\frac{4}{10}$ placed; $\frac{4}{10}$ circled; $>$
- Number line partitioned into sixths and thirds; $\frac{8}{6}$ and $\frac{5}{3}$ placed; $\frac{8}{6}$ circled; $<$
- Jay, explanations will vary.
- Wendy, explanations will vary.
- Number line partitioned into sixths, thirds, and halves; $\frac{5}{6}$, $\frac{5}{3}$, and $\frac{3}{2}$ placed; $\frac{5}{6} < \frac{3}{2} < \frac{5}{3}$; explanations will vary.

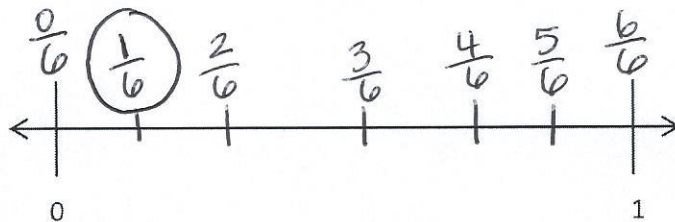
Homework Samples

Place the two fractions on the number line. Circle the fraction with the distance closest to 0. Then, compare using $>$, $<$, or $=$.

1. $\frac{1}{3}$ $<$ $\frac{2}{3}$



2. $\frac{4}{6}$ $>$ $\frac{1}{6}$



Lesson 19

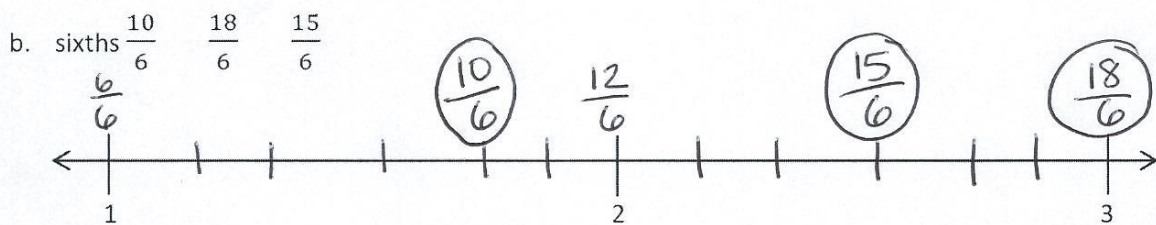
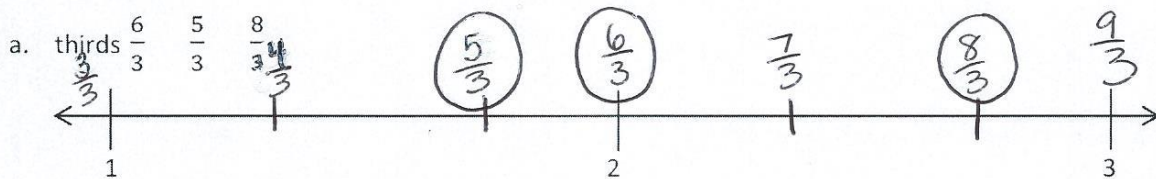
Objective: Understand distance and position on the number line as strategies for comparing fractions.

Homework Key

- Number line divided into thirds; given fractions placed; each whole written correctly as a fraction
 - Number line divided into sixths; given fractions placed; each whole written correctly as a fraction
 - Number line divided into fifths; given fractions placed; each whole written correctly as a fraction
- Row 1: $>$, $<$, $>$
Row 2: $=$, $<$, $>$
Row 3: $>$, $=$, $>$
- Answers will vary.
- Answers will vary.
- Answers will vary.

Homework Sample

1. Divide each number line into the given fractional unit. Then, place the fractions. Write each whole as a fraction.



Grade 3 Module 5 Topic E

Equivalent Fractions

Focus Standards:

- 3.NF.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
- Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
 - Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
 - Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.

Instructional Days Recommended: 8

In Topic D, students practiced placing and comparing fractions on a number line. In Topic E, they identify equivalent fractions using fraction strips, number bonds, and the number line as models. Students compare fractions on the number line to recognize that equivalent fractions refer to the same whole. They say $\frac{1}{2} = \frac{2}{4}$, assuming they are comparing like units (e.g. $\frac{1}{2}$ gallon = $\frac{2}{4}$ gallon rather than $\frac{1}{2}$ cup = $\frac{2}{4}$ gallon.) Likewise, when they model $\frac{1}{2} = \frac{2}{4}$ on the number line, both fractions are in reference to the same length unit, the same whole. Equivalent fractions are different ways to represent the same number, the same point on the number line. Initially, students find equivalence in fractions less than 1 whole (e.g., 1 half = 2 fourths). They then express whole numbers as fractions, using number bonds and number lines, to show how many copies of a unit are needed to make the whole (e.g., 4 copies of 1 fourth equals 1 whole). They reason about why whole numbers can be written as fractions with a denominator of 1. Finally, students explain equivalence through manipulating units.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 20

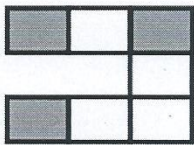
Objective: Recognize and show that equivalent fractions have the same size, though not necessarily the same shape.

Homework Key

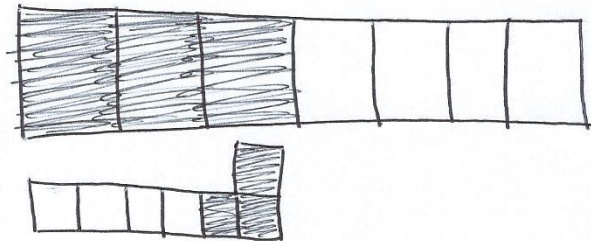
- $\frac{3}{7}$; two different representations of $\frac{3}{7}$ drawn
- No, these shapes are not equivalent. Although both shapes show $\frac{4}{5}$, the units are not the same size.
 - Two different representations of $\frac{4}{5}$ that are equivalent are drawn correctly.
- Neither, explanations will vary.

Homework Samples

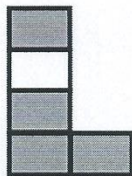
- Label the shaded fraction. Draw 2 different representations of the same fractional amount.



$$\frac{3}{7}$$



- These two shapes both show $\frac{4}{5}$.



- Are the shapes equivalent? Why or why not?

No, the shape on the right is bigger, so each equal part is bigger.

- Draw two different representations of $\frac{4}{5}$ that are equivalent.



Lesson 21

Objective: Recognize and show that equivalent fractions refer to the same point on the number line.

Homework Key

1. fourths: $\frac{0}{4}, \frac{3}{4}, \frac{5}{4}, \frac{6}{4}$; eighths: $\frac{2}{8}, \frac{4}{8}, \frac{7}{8}, \frac{8}{8}, \frac{12}{8}, \frac{16}{8}$

thirds: $\frac{1}{3}, \frac{3}{3}, \frac{5}{3}$; sixths: $\frac{1}{6}, \frac{4}{6}, \frac{6}{6}, \frac{8}{6}, \frac{10}{6}, \frac{12}{6}$

2. Shaded purple: $\frac{4}{4}, 1, \frac{8}{8}; \frac{3}{3}, 1, \frac{6}{6}$

Shaded yellow: $\frac{2}{4}, \frac{4}{8}$

Shaded blue: $\frac{8}{4}, 2, \frac{16}{8}; \frac{6}{3}, 2, \frac{12}{6}$

Shaded green: $\frac{5}{3}, \frac{10}{6}$

Answers will vary.

3. Row 1: 2; 8; 4

Row 2: 6; 6; 16

Homework Sample

1. Use the fractional units on the left to count up on the number line. Label the missing fractions on the blanks.



Lesson 22 - 23

Objective: Generate simple equivalent fractions by using visual fraction models and the number line.

Homework Key (22)

1. $\frac{1}{2}$ matched to $\frac{3}{6}$
 $\frac{2}{5}$ matched to $\frac{4}{10}$
 $\frac{8}{10}$ matched to $\frac{4}{5}$
 $\frac{2}{8}$ matched to $\frac{1}{4}$

2. 8, 6, 18

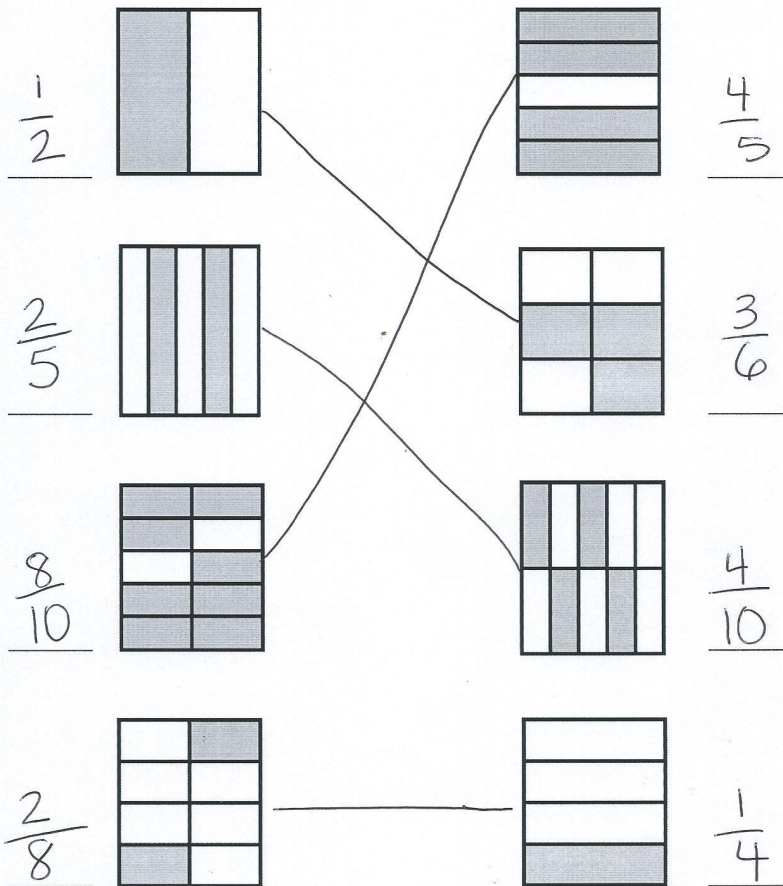
3. Explanations will vary.

4. 3 ninths, explanations will vary.

5. 6; explanations will vary.

Homework Sample

1. Write the shaded fraction of each figure on the blank. Then, draw a line to match the equivalent fractions.



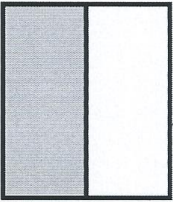
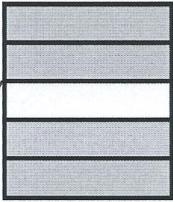
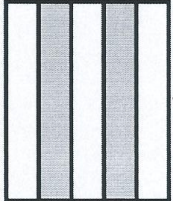
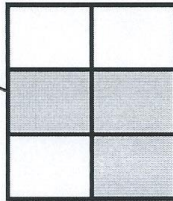
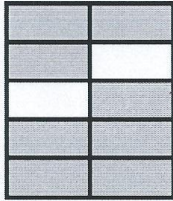
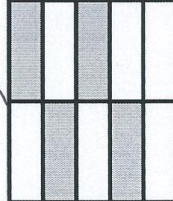
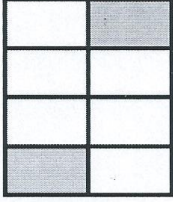
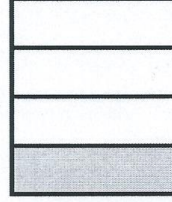
Lesson 23

Homework Key

- Number line divided into thirds and labeled correctly with a colored pencil
- Number line divided into sixths and labeled correctly with another colored pencil
- $\frac{0}{3} = \frac{0}{6}, \frac{1}{3} = \frac{2}{6}, \frac{2}{3} = \frac{4}{6}, \frac{3}{3} = \frac{6}{6}, \frac{4}{3} = \frac{8}{6}, \frac{5}{3} = \frac{10}{6}, \frac{6}{3} = \frac{12}{6}, \frac{7}{3} = \frac{14}{6}, \frac{8}{3} = \frac{16}{6}, \frac{9}{3} = \frac{18}{6}$
- 10; 24; number line drawn and labeled with appropriate fractions
- $\frac{2}{3} = \frac{4}{6}; \frac{1}{4} = \frac{2}{8}; \frac{7}{4} = \frac{14}{8}; \frac{7}{5} = \frac{14}{10}$
- $\frac{4}{12}$; number line drawn to explain the answer

Homework Sample

- Write the shaded fraction of each figure on the blank. Then, draw a line to match the equivalent fractions.

<u>$\frac{1}{2}$</u>			<u>$\frac{4}{5}$</u>
<u>$\frac{2}{5}$</u>			<u>$\frac{3}{6}$</u>
<u>$\frac{8}{10}$</u>			<u>$\frac{4}{10}$</u>
<u>$\frac{2}{8}$</u>			<u>$\frac{1}{4}$</u>

Lesson 24

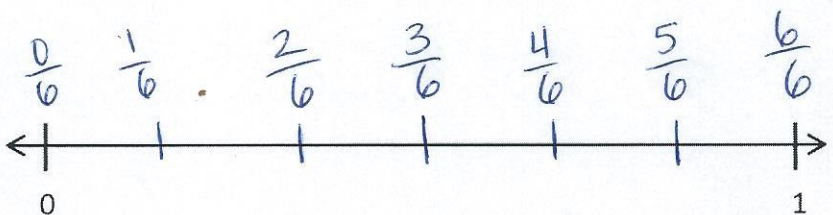
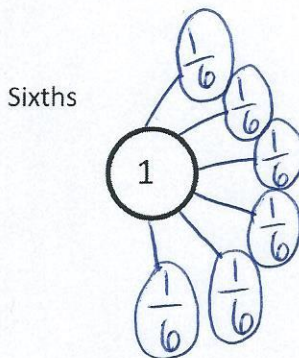
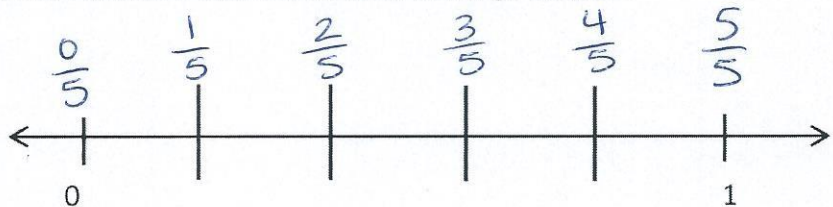
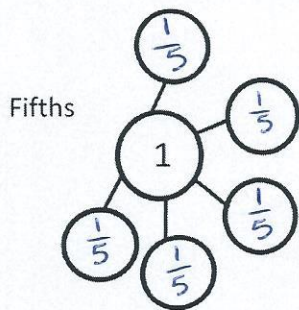
Objective: Express whole numbers as fractions and recognize equivalence with different units.

Homework Key

- Fifths: Number bond showing 5 units of $\frac{1}{5}$; number line partitioned and labeled from 0 to 1
Sixths: Number bond showing 6 units of $\frac{1}{6}$; number line partitioned labeled from 0 to 1
Sevenths: Number bond showing 7 units of $\frac{1}{7}$; number line partitioned and labeled from 0 to 1
Eighths: Number bond showing 8 units of $\frac{1}{8}$; number line partitioned and labeled from 0 to 1
- Fractions equal to 1 circled; $\frac{6}{6}, \frac{7}{7}, \frac{8}{8}$
- Answers will vary, $\frac{9}{9}$
- No, explanations will vary.

Homework Sample

- Complete the number bond as indicated by the fractional unit. Partition the number line into the given fractional unit, and label the fractions. Rename 0 and 1 as fractions of the given unit.



Lesson 25

Objective: Express whole number fractions on the number line when the unit interval is 1.

Homework Key

1. $\frac{4}{4}; \frac{4}{2}; \frac{4}{1}; \frac{8}{8}; \frac{8}{4}; \frac{8}{1}$

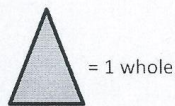
2. $\frac{0}{1}; \frac{4}{1}; \frac{6}{1}; \frac{12}{1}$

15, 16, $\frac{17}{1}; \frac{18}{1}; 19, \frac{20}{1}; \frac{21}{1}$

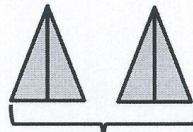
3. Explanations will vary.

Homework Sample

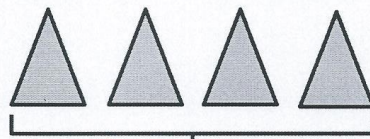
1. Label the following models as fractions inside the boxes.



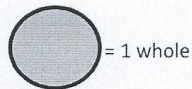
$$\frac{4}{4}$$



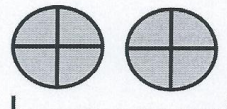
$$\frac{4}{2}$$



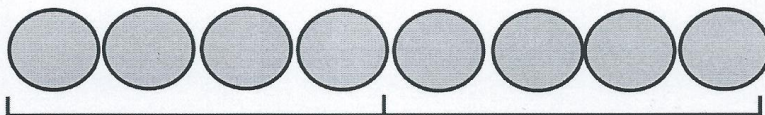
$$\frac{4}{1}$$



$$\frac{8}{8}$$



$$\frac{8}{4}$$



$$\frac{8}{1}$$

Lesson 26

Objective: Decompose whole number fractions greater than 1 using whole number equivalence with various models.

Homework Key

1. Sixths: 0, 0; 6, 6; 12; number bond completed

Fifths: 10, 10; 15, 15; 20, 20; number bond completed

2. Thirds: Answer provided

Sevenths: $\frac{14}{7}, \frac{21}{7}, \frac{28}{7}$

Eighths: $\frac{16}{8}, \frac{24}{8}, \frac{32}{8}$

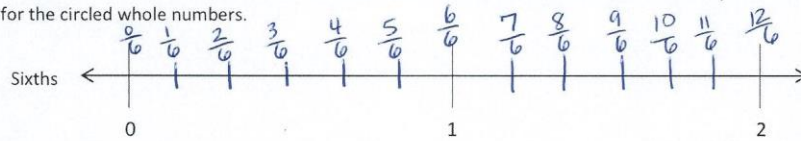
Tenths: $\frac{20}{10}, \frac{30}{10}, \frac{40}{10}$

3. Number line drawn to represent the basketball court, partitioned into thirds, and labeled correctly;

Day 1: $\frac{1}{3}$, Day 2: $\frac{2}{3}$, Day 3: $\frac{3}{3}, \frac{3}{3}$

Homework Sample

1. Partition the number line to show the fractional units. Then, draw number bonds with copies of 1 whole for the circled whole numbers.



0 = 0 sixths

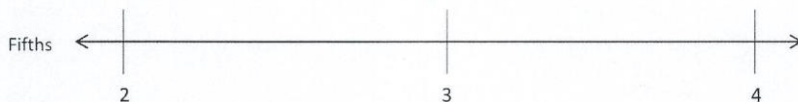
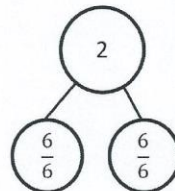
1 = 6 sixths

2 = 12 sixths

0 = $\frac{0}{6}$

1 = $\frac{6}{6}$

2 = $\frac{12}{6}$



2 = fifths

3 = fifths

4 = fifths

2 = $\frac{\square}{5}$

3 = $\frac{\square}{5}$

4 = $\frac{\square}{5}$



Lesson 27

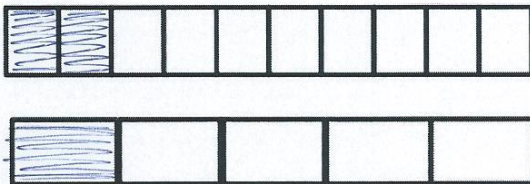
Objective: Explain equivalence by manipulating units and reasoning about their size.

Homework Key

- 1, 1, bigger
3, 3, smaller
- $\frac{1}{4}$ of a pizza, $\frac{1}{4} = \frac{2}{8}$
- Explanations will vary.
- 2 eighths; model drawn to support the answer
- 16 slices; explanations will vary.

Homework Sample

- Use the pictures to model equivalent fractions. Fill in the blanks, and answer the questions.



2 tenths is equal to 1 fifths.

$$\frac{2}{10} = \frac{1}{5}$$

The whole stays the same.

What happened to the size of the equal parts when there were fewer equal parts?

The equal parts were bigger.



1 third is equal to 3 ninths.

$$\frac{1}{3} = \frac{3}{9}$$

The whole stays the same.

What happened to the size of the equal parts when there were more equal parts?

The equal parts were smaller pieces.

Grade 3 Module 5 Topic E

Comparison, Order, and Size of Fractions

Focus Standards:

- 3.NF.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
- d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Instructional Days Recommended: 3

Fraction strips and the number line carry into Topic F as students compare fractions with the same numerator. As they study and compare different fractions, students continue to reason about their size. They develop the understanding that the numerator or number of copies of the fractional unit (shaded parts) does not necessarily determine the size of the fraction. The module closes with an exploration in which students are guided to develop a method for precisely partitioning various wholes into any fractional unit, using the number line as a measurement tool.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 28


Objective: Compare fractions with the same numerator pictorially.

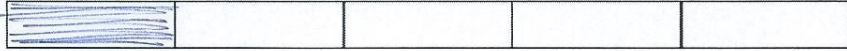
Homework Key

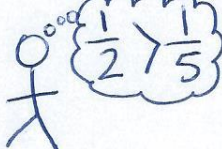
1. Models shaded correctly; 1 half circled
2. Models shaded correctly; 2 fourths circled
3. Models shaded correctly; 4 fifths circled
4. Models shaded correctly; 5 sevenths circled
5. Models are shaded correctly; 4 fourths circled
6. Saleem; tape diagrams drawn correctly
7. Lily; tape diagrams drawn correctly

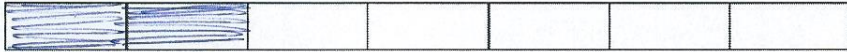
Homework Samples


Shade the models to compare the fractions. Circle the larger fraction for each problem.

1. 1 half 

1 fifth 



2. 2 sevenths 

2 fourths 

Lesson 29

Objective: Compare fractions with the same numerator using $<$, $>$, or $=$, and use a model to reason about their size.

Homework Key

1. $\frac{5}{12} < \frac{5}{6}$

2. $\frac{6}{6} > \frac{6}{12}$

3. $\frac{2}{4} > \frac{2}{9}$

4. $\frac{1}{2} > \frac{1}{6}$

5. a. $<$

b. $<$

c. $>$

6. Models drawn correctly; $<$

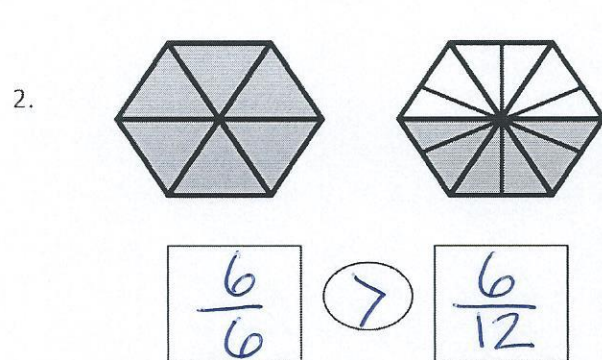
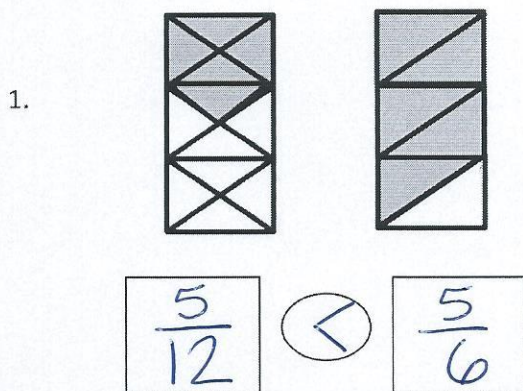
7. Models drawn correctly; $>$

8. Michello; models drawn correctly

9. Jahsir; models drawn correctly

Homework Samples

Label each shaded fraction. Use $>$, $<$, or $=$ to compare.



Lesson 30

Objective: Partition various wholes precisely into equal parts using a number line method.

Homework Key

Answers will vary.

Homework Sample

Describe step by step the experience you had of partitioning a length into equal units by simply using a piece of notebook paper and a straight edge. Illustrate the process.

